1st Christmas Junior Math Olympiad (Day 1)

January 8–29, 2018

CJMO 1. Determine the greatest positive integer m with the following property: it is possible to select integers $1 \le A_1 < A_2 < \cdots < A_m \le 2017$ such that there are no indices $1 \le i, j \le m$ with $i - j \ge 2$ and $i - j \mid A_i - A_j$.

CJMO 2. Let ABC be a triangle with centroid G and incenter I. Let B' and C' be the tangency points between the incircle and sides AC and AB, respectively, and let M and N be the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively. Let B'' be the reflection of B' across N and C'' the reflection of C' across M. Let X, Y, Z be the intersections of lines BB'' and CC'', lines NI and CC'', and lines MI and BB'', respectively.

(a) Show that $IYXZ$ is a parallelogram.

(b) Show that $[BIG] + [CIG] = [MGX] + [NGX]$.

CJMO 3. Let a, b, and c be pairwise distinct positive integers, and let r be the number of distinct primes dividing c.

(a) Show that there are at most $r + 1$ nonnegative integers k such that $a^{3^k} + b^{3^k} \mid c^{3^k}$.

(b) Find all such triples (a, b, c) for which $a + b$ is not a multiple of 3, and $a^k + b^k \mid c^k$ for all $k = 1, 2, \ldots, r + 1$.

Time: 4 hours and 30 minutes.
Each problem is worth 7 points.