**CMC**

CHRISTMAS MATHEMATICS COMPETITIONS

## American Mathematics Competitions

*The problems and solutions for this AMC 10 were prepared by the problem writing Committee for the mock AMC 10 and AMC 12:*


atmchallenge

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illogical_21

Th3Numb3rTh33

tree3


## MOCK 2018 AIME

The 1st annual AIME will be released on Friday, December 22, with the alternate on Friday, December 29. It is a 15-question, 3-hour, integer-answer exam. All students are invited to participate. All students are also invited to take the USA Mathematical Olympiad (USAMO) which will be released on January 5, 2017. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

## PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at the MAA web site: maa.org/math-competitions

**CMC**

CHRISTMAS MATHEMATICS COMPETITIONS

American Mathematics Competitions

1st Annual

# AMC 10 A

**American Mathematics Contest 10 A**

**FRIDAY, December 22, 2017**

## INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

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The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

*All students will be invited to take the 1st annual American Invitational Mathematics Examination (AIME) released on Friday, December 22, 2017 or Friday, December 29, 2017. More details about the AIME and other information are on the back page of this test booklet.*

1. What is the value of $(2+0+1)^7 - 201 + 7 \times (2+0) + 17$?

**(A)** 559 **(B)** 1983 **(C)** 1989 **(D)** 2008 **(E)** 2017

2. In how many ways can we arrange the numbers $1, 2, 3, 4$ in a line if the sum of two adjacent numbers is always prime?

**(A)** 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

3. Inside a bag containing geometric shapes, there are twice as many equilateral triangles with side $l$ as squares with side 3. If the sum of the areas of all the squares is equal to the sum of the areas of all the equilateral triangles, find the value of $l$.

**(A)** $\sqrt{6}$ **(B)** 3 **(C)** $\frac{16}{5}$ **(D)** $\sqrt[4]{108}$ **(E)** $2\sqrt{3}$

4. Let $\triangle ABC$ be an equilateral triangle, and let $D$ be a point such that $B$ is the midpoint of $AD$. If $CD = 12$, the area of $\triangle ABC$ can be expressed in the form $a\sqrt{b}$ for some integers $a, b$, where $b$ is not divisible by the square of any prime. Find $a + b$.

**(A)** 9 **(B)** 15 **(C)** 21 **(D)** 24 **(E)** 27

5. For what value of $a$ does the following system of equations

$$-20x + 17y = 2017$$
$$(-20)^2 x - ay = (2017)^2$$

have no real solutions $(x, y)$?

**(A)** $-340$ **(B)** $-289$ **(C)** 0 **(D)**289 **(E)** 340

6. There exists unique digits $a \neq 0$ and $b \neq a$ such that the four-digit number $\overline{aabb}$ is a perfect square. Compute $a + b$.

**(A)** 4 **(B)** 7 **(C)** 8 **(D)** 11 **(E)** 14

7. How many ordered pairs $(x, y)$ of integers satisfy

$$x^4 + y^4 + 2x^2y^2 + 1023 = 1024x^2 + 1024y^2 \ ?$$

**(A)** 4 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** 20

# 2018

# AMC 10 A

## DO NOT OPEN UNTIL FRIDAY, December 22, 2017

## **Administration On An Earlier Date Will Disqualify Your Schools Results**

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE DECEMBER 22, 2017. Nothing is needed from inside this package until DECEMBER 22.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be submitted through the Google Form provided no later than December 29.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

*The*

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23. A lattice point is a point in the coordinate plane, whose coordinates are both integers. There exists exactly 2017 lattice points that lie inside or on the region enclosed by the lines $x = 0, y = 0$, and $3x + 4y = 4a$, for some positive integer $a$. How many of these lattice points lie strictly inside this region?

**(A)** 1873 **(B)** 1909 **(C)** 1921 **(D)** 1945 **(E)** 1963

24. In $\triangle ABC$, let the bisector of $\angle BAC$ meet $BC$ and the circumcircle of $\triangle ABC$ at points $D$ and $E$, respectively, and let $F$ be the midpoint of $BC$. If $[DEF] = 1/3$, $DF = 1/2$, and $BC = 5$, what is the area of $\triangle ABC$?

**(A)** $\frac{197}{20}$ **(B)** $\frac{720}{73}$ **(C)** $\frac{15\sqrt{7}}{4}$ **(D)** $\frac{289}{27}$ **(E)** $\frac{16\sqrt{7}}{3}$

25. A $M \times N$ grid of unit squares is tiled with as many $5 \times 1$ indistinguishable rectangles as possible, such that every tile lies strictly inside the grid and no tiles overlap, leaving exactly one $1 \times 1$ unit square. If there are exactly 100 positions this square can occupy, what is the sum of all possible distinct values of $M + N$?

**(A)** 2030 **(B)** 2110 **(C)** 2190 **(D)** 2270 **(E)** 2350



8. Let $f_1(x) = |x - 2018|$ and define $f_{n+1}(x) = f_1(f_n(x))$. Compute

$$f_1(1) - f_2(2) + f_3(3) - f_4(4) + \cdots - f_{2018}(2018).$$

**(A)** $-2018$ **(B)** $-2017$ **(C)** $-1009$ **(D)** $-1008$ **(E)** 0

9. Call a positive integer *super-even* if the last two digits of all its multiples are always even. If $S$ is the sum of all super-even integers $1 \leq n \leq 2018$, what is the number of factors of $S$?

**(A)** 12 **(B)** 18 **(C)** 24 **(D)** 32 **(E)** 36

10. In $\triangle ABC$, $AC = 3, AB = 4$, $BC = 5$, and $I$ is the incenter of the triangle. Extend $BI$ and $CI$ to points $D$ and $E$, respectively, such that $BE \perp BI$ and $CD \perp CI$. What is the ratio between the areas of $\triangle CDI$ and $\triangle BEI$?

**(A)** $\frac{1}{2}$ **(B)** $\frac{9}{16}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{3}$ **(E)** $\frac{16}{9}$

11. Let $p, q$, and $r$ be distinct prime numbers such that $p^2+1$, $q^2+2$, and $r^2 + 3$, form an arithmetic sequence in that order. What is the minimum possible value of $p + q + r$?

**(A)** 15 **(B)** 25 **(C)** 30 **(D)** 37 **(E)** 40

12. Three triplets, Tam, Tom, and Tim, work together to build a treehouse. Tam works twice as fast as Tom, but three times slower than Tim. After working together for 10 days, Tim decides to quit since he is tired. After another 10 days, Tom also decides to quit. It takes Tam another 6 days to complete the treehouse. The number of days it would had taken them to complete the treehouse if none of them had quit can be written as $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$. What is $m + n$?

**(A)** 22 **(B)** 47 **(C)** 53 **(D)** 66 **(E)** 91

13. What is the sum of all integers $1 \leq m \leq 100$ such that there exists an integer $n$ which has the property that exactly $m\%$ of the positive integer divisors of $n$ are perfect squares?

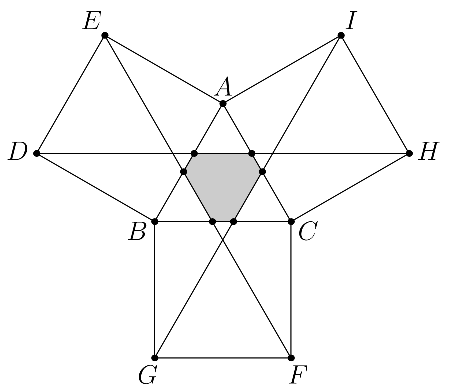
**(A)** 321 **(B)** 343 **(C)** 373 **(D)** 421 **(E)** 434

14. Suppose that the graphs of $y = x^2 - 4x - 7$ and $x = (y + a)^2$ intersect at an odd number of points. What is the nearest integer to the sum of all possible values of $a$?

**(A)** 28 **(B)** 29 **(C)** 30 **(D)** 31 **(E)** 32

15. Equilateral triangle $\triangle ABC$ has $AB = BC = AC = 1$. Squares $ABDE$, $BCFG$, and $ACHI$ are constructed outside $\triangle ABC$ with sides coinciding with $AB$, $BC$, and $AC$, respectively. Connect $EF$, $GI$, and $DH$, so that they intersect $\triangle ABC$ at six points, forming a hexagon. Then, the area of the hexagon can be expressed in the form $\frac{a-b\sqrt{c}}{d}$ for some positive integers $a, b, c, d$, where $c$ is not divisible by the square of any prime and $\gcd(a, b, d) = 1$. Compute $a+b+c+d$.

**(A)** 10 **(B)** 12 **(C)** 16 **(D)** 22 **(E)** 26




16. A positive integer is called *triangular number* if it can be expressed in the form $\frac{n(n+1)}{2}$ for a positive integer $n$. How many ordered pairs $(x, y)$ of triangular numbers satisfy $x - y = 31,500$?

**(A)** 12 **(B)** 18 **(C)** 24 **(D)** 36 **(E)** 48

17. Consider all positive integers $n$, with the property that $35 \times n$ has exactly 3500 digits. Let $N$ be the largest integer with such property. What is the sum of the digits of $N$?

**(A)** 15743 **(B)** 15751 **(C)** 15752 **(D)** 15753 **(E)** 15761



18. Consider quadrilateral $ABCD$ with $AB = 3$, $BC = 4$, $CD = 5$, and $DA = 6$. Circle $\Gamma$ passes through $A, B, C$ and intersects $DA$ again at $X$. Given that $CX = 4$, compute the area of quadrilateral $ABCD$.

**(A)** 18 **(B)** $6\sqrt{10}$ **(C)** 19 **(D)** $11\sqrt{3}$ **(E)** $8\sqrt{6}$

19. Define $f(n)$ to be the sum of the positive integer factors of $n$. If $f(n) = 360$ and $f(3n) = 1170$, what is the smallest possible value of $f(2n)$?

**(A)** 720 **(B)** 729 **(C)** 744 **(D)** 756 **(E)** 840

20. Miriam is trying to escape from a haunted house placed in the middle of a circular field with area 2018 square feet. Each minute, she carefully moves 1 foot in any direction she wants; however, an evil ghost may decide to scare her and make her run 1 foot in the direction opposite to the one she wanted. What is the least integer number of feet Miriam must walk in order to escape from the field, independently from the ghost's tricks?

**(A)** 623 **(B)** 632 **(C)** 643 **(D)** 672 **(E)** 673

21. Eddie has a birthday party, in which he invited his 8, very generous, best friends. Each friend brought a certain number of gifts for Eddie. It is observed that for every group of 4 friends, at least 2 of them brought the same number of gifts. Moreover, the person with the most gifts brought 4, while the least brought 1. What is the total number of possible values of the amount of gifts Eddie received in total?

**(A)** 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

22. In a plane lies 14 points and all possible lines are drawn among themselves, forming 287 triangles. How many quadrilaterals are formed?

**(A)** 308 **(B)** 364 **(C)** 383 **(D)** 409 **(E)** 441