2020 CMC ARML Individual Questions 1 and 2
(10 minutes)

Name: ____________________________

Team: ____________________________

Answer to I-1: [ ]   Answer to I-2: [ ]

I-1. Compute the maximum value of n for which n cards, numbered 1 through n, can be arranged and lined up in a row such that

- it is possible to remove 20 cards from the original arrangement leaving the remaining cards in ascending order, and
- it is possible to remove 20 cards from the original arrangement leaving the remaining cards in descending order.

I-2. Let $ABCD$ be a quadrilateral with side lengths $AB = 2$, $BC = 5$, $CD = 3$, and suppose $\angle B = \angle C = 90^\circ$. Let M be the midpoint of $\overline{AD}$ and let P be a point on $\overline{BC}$ so that quadrilaterals $ABPM$ and $DCPM$ have equal areas. Compute PM.

2020 CMC ARML Individual Questions 3 and 4
(10 minutes)

Name:	____________________		
Team:	____________________		
Answer to I-3:		Answer to I-4:	

I-3. There is a unique nondecreasing sequence of positive integers a_1, a_2, …, a_n such that

$$\left(a_1 + \frac{1}{a_1}\right)\left(a_2 + \frac{1}{a_2}\right)\cdots\left(a_n + \frac{1}{a_n}\right) = 2020.$$

Compute $a_1 + a_2 + \cdots + a_n$.

I-4. Let $0^\circ < \theta < 90^\circ$ be an angle. If

$$\log_{\sin\theta}\cos\theta, \quad \log_{\cos\theta}\tan\theta, \quad \log_{\tan\theta}\sin\theta$$

form a geometric progression in that order, compute $\sin\theta$.

2020 CMC ARML Individual Questions 5 and 6
(10 minutes)

Name: ______________________

Team: ______________________

Answer to I-5: [ ] Answer to I-6: [ ]

I-5. Let ABC be a triangle and let M be the midpoint of $\overline{BC}$. The lengths AB, AM, AC form a geometric sequence in that order. The side lengths of $\triangle ABC$ are 2020, 2021, x in some order. Compute the sum of all possible values of x.

I-6. Let $\mathcal{C} = \{(x, y, z) : 0 \le x, y, z \le 1\}$. Real numbers a, b, c are selected randomly and independently such that $0 < a, b, c < 1$. Given that $\mathcal{C}$ and the plane $ax+by+cz = 1$ intersect, compute the probability that their intersection is a nondegenerate hexagon.

2020 CMC ARML Individual Questions 7 and 8
(10 minutes)

Name:	______________________		
Team:	______________________		
Answer to I-7:		Answer to I-8:	

I-7. Compute

$$21\left(1+\frac{20}{2}\left(1+\frac{19}{3}\left(1+\frac{18}{4}\left(\cdots\left(1+\frac{12}{10}\right)\cdots\right)\right)\right)\right).$$

I-8. Let ABC be an equilateral triangle with circumcircle ω. Select a point P on the minor arc BC of ω such that the distance from P to line AB is 1, and so that the distance from P to line AC is 2. Compute the side length of $\triangle ABC$.

2020 CMC ARML Individual Questions 9 and 10
(10 minutes)

Name:	______________________		
Team:	______________________		
Answer to I-9:		Answer to I-10:	

I-9. Let $\lceil x \rceil$ denote the smallest integer greater than or equal to x. The sequence (a_i) is defined as follows: $a_1 = 1$, and for all $i \geq 1$,

$$a_{i+1} = \min\left\{7\left\lceil \frac{a_i + 1}{7} \right\rceil, 19\left\lceil \frac{a_i + 1}{19} \right\rceil\right\}.$$

Compute a_{100}.

I-10. Let $S(n)$ denote the sum of the digits of a positive integer n. Compute the number of positive integers n for which

- n only has nonzero digits, and
- $(S(2n))^2 + 2S(n) + 1 = 345$.