Christmas Math Competitions

2020 CMC ARML Relay Questions

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R1-1. Let n be a two-digit integer, and let m be the result when we reverse the digits of n. If $n - m$ and $n + m$ are both perfect squares, find n.

R1-2. Let $T = \text{TNYWR}$. Arrange the numbers 0, 1, 2, ..., T in a circle. What is the expected number of (unordered) pairs of adjacent numbers that sum to T?

R1-3. Let T = TNYWR. Let $ABCD$ be a parallelogram with area 2020 such that $AB/BC = T$. The bisectors of $\angle DAB$, $\angle ABC$, $\angle BCD$, $\angle CDA$ form a quadrilateral. Compute the area of this quadrilateral.

R2-1. Compute the number of ordered triples (p, q, r) of primes, each at most 30, such that

$$p + q + r = p^2 + 4.$$

R2-2. Let $T = \text{TNYWR}$. Let ABC be a triangle with incircle ω. Points E, F lie on $\overline{AB}$, $\overline{AC}$ such that $\overline{EF} \parallel \overline{BC}$ and $\overline{EF}$ is tangent to ω. If $EF = T$ and $BC = T + 1$, compute $AB + AC$.

R2-3. Let $T = \text{TNYWR}$. There is a positive integer k such that T is the remainder when $17^0 + 17^1 + 17^2 + \cdots + 17^k$ is divided by 1000. Compute the remainder when 17^k is divided by 1000.