

The *AMC 12* consists of 25 problems, where for each problem, a correct answer is worth 6 points, leaving the problem blank is worth 1.5 points, and an incorrect answer is worth 0 points. The *AIME* consists of 15 problems, where each problem is worth 10 points, and no partial credit is given.

Any contestant who scores at least 84 on the AMC 12 is eligible for the AIME, and the USAMO index of such a student is the sum of his AMC 12 and AIME scores. Determine the smallest integer $N > 300$ such that no contestant can possibly obtain a USAMO index of $\frac{1}{2}N$.