2019 CMC ARML Individual Questions 1 and 2
(10 minutes)

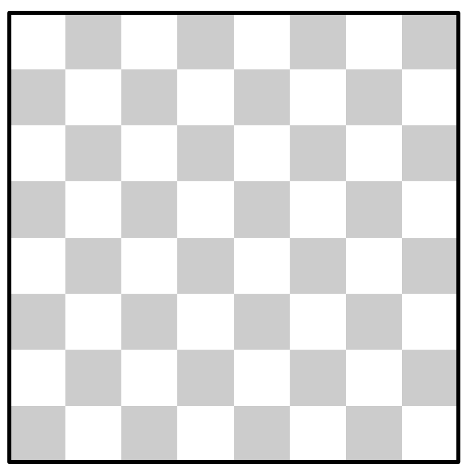
Name: ____________________________

Team: ____________________________

Answer to I-1: [   ] Answer to I-2: [   ]

I-1. In triangle ABC, $\angle A = 60^\circ$ and $BC = 12$. Given that the perimeter of $\triangle ABC$ is 30, compute the area of $\triangle ABC$.

I-2. Consider the set of rectangles formed by the unit squares of an 8×8 checkerboard. Compute the number of these rectangles contain exactly four black squares.



2019 CMC ARML Individual Questions 3 and 4
(10 minutes)

Name: ____________________________

Team: ____________________________

Answer to I-3: [ ] Answer to I-4: [ ]

I-3. Compute $\dfrac{3^{\log_{2/3} 3}}{2^{\log_{2/3} 2}}$.

I-4. The numbers $a_1, a_2, a_3, \ldots$ form an arithmetic sequence. Suppose that there exist unique positive integers $p < q < 63$ such that $a_p = p^2$, $a_q = q^2$, and $a_{63} = 2019$. Compute $p + q$.

2019 CMC ARML Individual Questions 5 and 6
(10 minutes)

Name: ______________________

Team: ______________________

Answer to I-5: [ ]   Answer to I-6: [ ]

I-5. Each second, two randomly chosen digits of 123 are swapped. Compute the expected number of seconds until the number 321 is formed.

I-6. Let $\triangle ABC$ be an isosceles triangle with $AB = AC = 1$, and let E and F be the feet of the altitudes from B and C to sides $\overline{AC}$ and $\overline{AB}$, respectively. If line EF is tangent to the incircle of $\triangle ABC$, compute the perimeter of $\triangle ABC$.

2019 CMC ARML Individual Questions 7 and 8
(10 minutes)

Name: ________________________

Team: ________________________

Answer to I-7: ☐ Answer to I-8: ☐

I-7. In acute triangle ABC, $AB = 2$ and $AC = 3$. Let D be the foot of the angle bisector of $\angle BAC$. The circle with diameter AD intersects $\overline{AB}$ and $\overline{AC}$ at E and F, respectively. Given that the area of $\triangle DEF$ is one sixth of the area of $\triangle ABC$, compute BC^2.

I-8. There exist unique positive integers $1 < a < b$ satisfying

$$a^2 + b^2 = 2^{20} + 1.$$

Compute $a + b$.

2019 CMC ARML Individual Questions 9 and 10
(10 minutes)

Name: ______________________

Team: ______________________

Answer to I-9: [ ] Answer to I-10: [ ]

I-9. Let a and b be real numbers such that the polynomial

$$P(x) = x^3 - ax^2 + 11ax - b$$

has three distinct real roots forming a geometric progression. Compute b.

I-10. Isabel and Robert play a game in which Isabel chooses a list of 6 real numbers and Robert devises 6 questions, each of the form "What is the sum of the x^{th} and y^{th} number in your list?" where $1 \leq x < y \leq 6$. Without regard to order, compute the number of ways can Robert choose his questions so that he can determine Isabel's numbers.