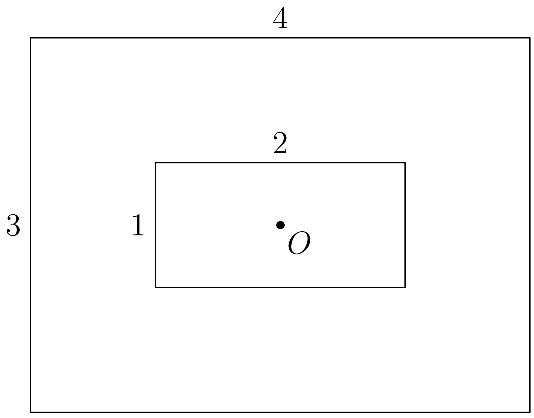
⋆ NATIONAL LEVEL ⋆

The Mandelbrot Competition

Round Two Test

Name: ____________________

Time Limit: 40 minutes

1. A vertical line intersects each of the graphs $y = \|x\|$ and $y = (x-5)(6-x)$ at a single point. What is the smallest possible distance between these two points?		①
2. Farmer Mandelbrot wants to construct a grazing field for his sheep bounded by a polygon-shaped fence. However, every night a hungry wolf comes to the fence at a random spot and hops on top of it. If he can see the entire field from his position, he will attack the sheep (where he cannot see through the fence). Otherwise, he fears that the remaining sheep will reveal his identity, so he will not attack. Compute the minimum number of sides Farmer Mandelbrot needs for his fence so that his sheep will not be attacked.		①
3. Consider an equilateral triangle grid of side length 2018, divided into unit equilateral triangles with lines parallel to the sides. Let $f(n)$ be the number of equilateral triangles $\triangle ABC$ with side length n and all sides on the triangular grid such that there exists exactly one point D on the grid making $ABCD$ a rhombus. Find the largest $n \leqslant 2018$ so that $f(n) > f(5)$.		②
4. In right triangle $\triangle ABC$, the area is twice the perimeter and all sides have integer lengths. Compute the sum of the lengths of all possible circumradii of $\triangle ABC$.		②
5. The diagram given depicts a 1×2 rectangle and a 3×4 rectangle centered at a common point O. Points F and G are selected on the boundary of the inner and outer rectangle, respectively. What is the maximum possible area of $\triangle FOG$? 		②

6. Let N_k, where k is a positive integer, denote the smallest positive integer with the property that there exists exactly k pairs of non-negative integers (a, b) satisfying $N = a^2 - b^2$. Find the smallest possible even value of N_k.		③
7. For $1 \leqslant m \leqslant 2018$, define $R(m)$ to be remainder when 2019 is divided by m, and let $\sigma(n)$ be the sum of the positive factors of n. Given that $R(1) + R(2) + \cdots + R(2018) = 723356$, compute the remainder when $\sigma(1) + \sigma(2) + \cdots + \sigma(2018)$ is divided by 1000.		③