



★ REGIONAL LEVEL ★

The Mandelbrot Competition

Round Two Test

Name: _____

*Time Limit:
40 minutes*

<p>1. A vertical line intersects each of the graphs $y = x$ and $y = (x - 5)(6 - x)$ at a single point. What is the smallest possible distance between these two points?</p>		①
<p>2. Farmer Mandelbrot wants to construct a grazing field for his sheep bounded by a polygon-shaped fence. However, every night a hungry wolf comes to the fence at a random spot and hops on top of it. If he can see the entire field from his position, he will attack the sheep (where he cannot see through the fence). Otherwise, he fears that the remaining sheep will reveal his identity, so he will not attack. Compute the minimum number of sides of Farmer Mandelbrot needs for his fence so that his sheep will not be attacked.</p>		①
<p>3. Let (a, b, c, d) be a randomly chosen permutation of $(1, 2, 3, 4)$. What is the probability that $a^b + c^d$ is a prime number?</p>		②
<p>4. Consider the equilateral triangle grid of side length 2018, where it is divided into unit equilateral triangles with lines parallel to the sides. Let $f(n)$ be the number of equilateral triangles ABC of length n with all sides on the triangular grid such that there exists exactly one point D with $ABCD$ being a rhombus on the grid. Find the largest $n \leq 2018$ so that $f(n) > f(5)$.</p>		②
<p>5. Let \mathcal{P}_n be the percentage of right triangles obtained by randomly choosing 3 of the vertices of a regular polygon with n sides. For how many n is \mathcal{P}_n a positive integer?</p>		②
<p>6. In right triangle ABC, the area is twice the perimeter and all sides have integer lengths. Compute the sum of all possible circumradii of ABC.</p>		③
<p>7. Let N_k, where k is a positive integer, denote the smallest positive integer with the property that there exists exactly k pairs of non-negative integers (a, b) satisfying $N = a^2 - b^2$. Find the smallest possible even value of N_k.</p>		③