⋆ National Level ⋆

The Mandelbrot Competition

Round One Test

Name: ____________________

Time Limit: 40 minutes

1. In the addition $\begin{array}{cccccc} & & T & I & D & E \\ + & & P & O & D & S \\ \hline & T & A & S & T & Y \end{array}$ each letter represents a distinct digit. Find the maximum possible value of $TASTY$.		(1)
2. The ordered triple $(33, 34, 35)$ is the smallest triple of consecutive integers, each with exactly 4 positive integer divisors. Find the next ordered triple with such property. Write your answer in the form $(n, n+1, n+2)$.		(1)
3. The set of points $S = \{P(x^2, y^2), x, y \in Z\}$ is plotted on the coordinate plane. How many squares with sides parallel to the coordinate axes have their four vertices in S and area less than 2018?		(2)
4. Let $\triangle ABC$ be an equilateral triangle with side length 1, and let $\mathcal{T}$ be the locus of all points P such that exactly one triangle in the set $\{\triangle APB, \triangle BPC, \triangle CPA\}$ is obtuse. The points in $\mathcal{T}$ form multiple regions of finite area. Find the total area of these regions.		(2)
5. Compute $\mathcal{S} = \sum_{a+b+c=12} ab + bc + ac$, where a, b, c are positive integers.		(2)
6. A sequence is initially $1, 2, \cdots, 13$. We perform an *operation* such that each number n of such sequence is replaced with $1, 2, \cdots, n$. Compute the average of the numbers in the sequence after 37 *operations*.		(3)
7. A semicircle with diameter $\overline{DE}$ is inscribed inside $\triangle ABC$, with $\overline{DE}$ on $\overline{AC}$ such that $AD = 24.5, CE = 25$, and $ED = 24$. Find the minimum possible area of $\triangle ABC$.		(3)