



★ NATIONAL LEVEL ★

The Mandelbrot Competition

Round Two Test

*Time Limit:
40 minutes*

Name: _____

<p>1. Ana and Banana are playing a game with a pile of n coins. Each turn, starting from Ana, each player removes either 1, 3 or 4 coins from the pile, and the player removing the last coin wins. For how many values of $1 \leq n \leq 2019$ is there a winning strategy for Banana?</p>		①
<p>2. A number n is reversible if reversing its digits then squaring it will result in the same number as squaring n then reversing its digits. For example, 1 and 2 are reversible numbers. What is the sum of all 2 digits reversible numbers that aren't a multiple of 10?</p>		①
<p>3. Illogical left his house $x < 12$ hours after noon, and when he came back $y < 12$ hours later, the hour and minute hands have switched positions. If $\lfloor x \rfloor$ is an even positive integer not divisible by 4 and $\lfloor y \rfloor$ is an odd positive integer, find the number of possible distinct arrival times.</p>		②
<p>4. A solid cylinder has radius 12 and height 7. It is sliced in half by a cutting plane that is tangent to both of its circular bases. What is the area of the cross section?</p>		②
<p>5. Find the number of ordered pairs of relatively prime positive integers (a, b) so that the equation $\lfloor ax \rfloor + \lfloor bx \rfloor = 2019$ has no solution for x over the real numbers.</p>		②
<p>6. Let \mathcal{C} be a cube with side length 1. Find the smallest possible integer value of n such that the surface area of the locus of all points whose sum of the square of the distances from the vertices of \mathcal{C} is n is at least 1000.</p>		③
<p>7. Caesar is playing with a row of 10,000 tin soldiers. Each time, he removes from the row all soldiers whose order in the row is a perfect square. He then puts the remaining soldiers in another row, repeating the process again. If the last soldier remaining was the n^{th} soldier in the original ordering, find n.</p>		③